MATHEMATICS (CORE)

1. **STANDARD OF THE PAPER**

The Chief Examiner reported that the standard of the paper compared favourably with that of the previous year.

2. **PERFORMANCE OF THE CANDIDATES**

The Chief Examiner reported that the performance of candidates declined as compared to that of last year.

3. **SUMMARY OF CANDIDATES' STRENGTHS**

The Chief Examiner listed some strengths of candidates as ability to:
(a) complete a table of values for a given quadratic relation within a given interval and draw the corresponding graph of the relation;
(b) complete tables for addition $\oplus$ and multiplication $\otimes$ in modulo 5;
(c) construct cumulative frequency table from a given data;
(d) solve problems involving transformation.

4. **SUMMARY OF CANDIDATES' WEAKNESSES**

The Chief Examiner listed some of the weakness of candidates as difficulty in:
(a) translating word problems into mathematical statements;
(b) solving problems on probability;
(c) solving equation simultaneously involving indices;
(d) solving problems involving mensuration;

5. **SUGGESTED REMEDIES**

The Chief Examiner suggested that teachers should give students sufficient exercises for the various topics treated. He also recommended that candidates should be given in-depth tuition in areas of their weaknesses by explaining thoroughly the relevant concepts.

6. **DETAILED COMMENTS**

Question 1

(a) Solve, correct to one decimal place, tan (θ +25°) = 5.145, where 0° $\leq \theta \leq$90°.

(b) In the relation $t = m\sqrt{n^2 + 4r}$:

(i) make n the subject of the relation;

(ii) find the positive value of n when t = 25, m = 5 and r = 4.

The candidates were required to find an acute angle from a given trigonometry equation in part (a). Most of the candidates equated (θ + 25°) to 5.145 instead of

$\theta + 25^\circ = \tan^{-1}(5.145)$

$\theta + 25^\circ = 79.00^\circ$

$\theta = 54.0^\circ$

In part (b), candidates were expected to make ***n*** the subject from a given relation and find the value of ***n*** given ***m***, ***t*** and ***r***. Most of the candidates who attempted this question did not perform well. A lot of candidates could not go beyond squaring both sides of the relation. Some candidates took the square root of only the first term $\frac{t}{m}$. The candidates were required to solve the (b) as

(b)(i) $t = m\sqrt{n^2 + 4r}$

$t^2 = m^2(n^2 + 4r)$

$t^2 = m^2n^2 + m^24$

$m^2n^2 = t^2 - m^24r$

$n^2 = \frac{t^2-m^24r}{m^2}$

$n = \sqrt{\frac{t^2-m^24r}{m^2}}$ or equivalent

(ii) $n = \sqrt{\frac{25^2-(5^2)(4\times4)}{5^2}}$

$n = \sqrt{\frac{625-(25\times16)}{25}} = \sqrt{\frac{625-400}{25}} = \sqrt{9}$

$n = 3$

Question 2

The first three terms of an Arithmetic Progression (A.P.) are (*x* + 1), (4*x* – 2) and (6*x* – 3) respectively. If the last term is 18, find the:

(a) value of *x*;

(b) sum of the terms of the progression.

The candidates were given three terms in an Arithmetic Progression to find the value of *x* in (a) and the sum of the terms of the sequence. In (a), candidates were required to use the concept of common difference of an A.P to find *x*. A lot of candidates did not know what to do to solve this problem. They were to use the concept 2nd term – 1st term = 3rd term – 2nd term. That is

(a) $(4x - 2) - (x + 1) = (6x - 3) - (4x - 2)$

$4x - 2 - x - 1 = 6x - 3 - 4x + 2$

$3x - 3 = 2x - 1$

$x = 2$

Most of the candidates could not find *x* in (a)

(b) Most of the candidates could not list the terms in the sequence given the last term and the question to be solved as:

first term = 2 + 1 = 3
second term = 8 – 2 = 6
third term = 12 – 3 = 9
18 = 3 + (n – 1)3
18 = 3 + 3n – 3
3n = 18
n = 6
Sum of the terms of the progression = $\frac{6}{2}(3 + 18)$
$= 3 \times 21$
= 63

<u>Question 3</u>

The angle of a sector of a circle with radius 22cm is 60°. If the sector is folded such that the straight edges coincide, forming a cone, calculate, correct to one decimal place, the:
(a) radius;
(b) height;
(c) volume;
of the cone. [Take $\pi = \frac{22}{7}$]

The candidates were expected to find the (a) radius, (b) height and (c) volume of a sector of a circle with radius 22cm and angle 60° that has been folded to form a cone.

Most of the candidates did not attempt this question although it was compulsory. The candidates were required to solve question (3) as

(a) Radius of the cone = $\frac{60^\circ}{360^\circ} \times 22$
= 3.6667 cm
≈ 3.7 cm (to **one** decimal place)

(b) Let h be the height of the cone
$h^2 = 22^2 - (3.6667)^2 = 470.5553$
h = 21.6923 cm
≈ 21.7 cm (to **one** decimal place)

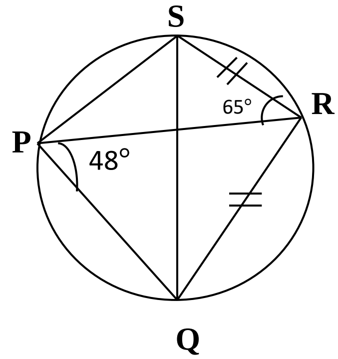
(c) Volume of the cone = $\frac{1}{3} \times \frac{22}{7} \times (3.6667)^2 \times 21.6923$
= 305.5341 cm^3
≈ 305.5 cm^3 (to **one** decimal place)

The few candidates who were able to solve this question approximated prematurely and hence obtained wrong answers at the final stage of the solution.

Question 4

(a) In how many years will GH¢312.50 invested at 4% per annum simple interest amount to GH¢500.00?

(b)



In the diagram, *PQRS* is a cyclic quadrilateral. If $|SR| = |RQ|$, $\angle SRP$ = 65° and $\angle RPQ$ = 48°find $\angle PRQ$.

In (a), a lot of candidates started solving without first finding the interest which is

Interest = GH¢ (500 – 312.50) = GH¢ 187.50

$187.5 = \frac{312.5 \times 4 \times n}{100}$

$1250n = 18750$

$n = 15$ years

In part (b), most candidates added 65° to 48° and subtracted the result from 180° without recourse to the given cyclic quadrilateral. That is

$\angle RSQ = \angle RPQ = 48^\circ$(Angles in the same segment are equal)

$\angle SQR = \angle RSQ = 48^\circ$(Base $\angle$s of Isosceles Δ QRS)

$48^\circ + 48^\circ + 65^\circ + \angle PRQ = 180^\circ$

$161^\circ + \angle PRQ = 180^\circ$

$\angle PRQ = 180^\circ - 161^\circ$

$\angle PRQ = 19^\circ$

Question 5

(a) The probabilities that James and Juliet will pass an examination are $\frac{3}{4}$ and $\frac{3}{5}$ respectively. Find the probability that both will fail the examination.

(b)

Balls	**Green**	**Blue**
New	**8**	**2**
Old	**4**	**6**

The table shows the distribution of balls in a bag. If 2 balls are selected at random with replacement, find the probability of selecting either 2 new green balls or 2 old blue balls.

Most of the candidates who answered this question performed poorly. Only a few candidates were able to answer the probability question in (a) correctly. In part (b), most of the candidates did not understand this question. From the table, there were (8+2) =10 new balls and (4+6) = 10 old balls. So, the total number of balls from which the selection is made are (10+10) = 20 balls. A lot of candidates wrongly considered 10 as the total number of balls. Also, since there was replacement before the next selection, the total number of ball at each selection is 20.
The candidates were required to solve this question as:

(a) P(James fail) $= \frac{1}{4}$ and P(Juliet) $= \frac{2}{5}$

P(**both** failed) $= \frac{1}{4} \times \frac{2}{5}$

$= \frac{1}{10}$ or 0.1

(b) P(new green) $= \frac{2}{5}$

P(old blue) $= \frac{3}{10}$

P(2 new green **or** 2 old blue)

$= \left(\frac{2}{5} \times \frac{2}{5}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$

$= \frac{4}{25} + \frac{9}{100} = \frac{16+9}{100}$

$= \frac{25}{100} = \frac{1}{4}$ or 0.25

Question 6

(a) If $9^x \times 3^{2y} = \frac{1}{729}$, and
$2^{-x} \times 4^{-y} = \frac{1}{8}$, find the values of *x* and *y*.

(b) Two commodities *X* and *Y* cost D70.00 and D80.00 per kg respectively. If 34.5kg of *X* is mixed with 26 kg of *Y* and the mixture is sold at D85.00 per kg, calculate the percentage profit.

The candidates were expected to apply laws of indices to derive two simultaneous equations and solve to obtain the values of *x* and *y* in (a). In part (b), candidates were required to calculate the profit of two commodities *X* and *Y*. Most of the candidates did not attempt this question. Those who attempted it did not perform well in (a). They could not apply the laws of indices properly.
Candidates were expected to solve this question as:

$3^{2x} \times 3^{2y} = 3^{-6} \Rightarrow 2x + 2y = -6$ ……..…………… (1)

$2^{-x} \times 2^{-2y} = 2^{-3} \Rightarrow -x - 2y = -3 \ldots$ ………………….. (2)

From equation (2), $x = 3 - 2y$.
Substituting $3 - 2y$ for x in equation (1)

$2(3 - 2y) + 2y = -6$

$6 - 4y + 2y = -6$

$-2y = -12$

$y = 6$

substituting for y and solving for x,

$x = 3 – 2(6) = 3 – 12$

$x = – 9$

In part (b), most of the candidates could not find the cost price of the mixture to enable them find the percentage profit. That is

Total cost of X = 70.00 × 34.5 =D 2415.00

Total cost of Y = 80.00 × 26 =D 2080.00

Total cost of mixture = 2415 + 2080 = D 4495.00

Total sales of mixture = 85 × (34.5+26) = 85 × 60.5

= D 5142.50

Profit = 5142.50 – 4495.00 = D 647.50

% profit = $\frac{647.5}{4495} \times 100\%$

$= 14.4049\ \%$

Candidates performance on this question was very poor.

Question 7

(a) Copy and complete the following table for the relation: $y = 2(x+2)^2 – 3$ for $-5 \leq x \leq 2$.

x	-5	- 4	-3	-2	-1	0	1	2
y			-1	-3		5		

(b) Using scales of 2cm to 1 unit on the x – axis and 2cm to 5 units on the y-axis, draw the graph of the relation $y = 2(x+2)^2 – 3$ for $-5 \leq x \leq 2$.

(c) Use the graph to find the solution of:

(i) $2(x + 2)^2 = 3$;

(ii) $2(x + 2)^2 = 5$.

(d) For what values of x, from the graph, is y increasing in the interval?

In parts (a) and (b), most of the candidates were able to complete the table for the relation $y = 2(x + 2)^2 – 3$ for $-5 \leq x \leq 2$ correctly and were able to draw the corresponding graph. However, some candidates did not use the given scales of 2cm to 1unit on the x – axis and 2cm to 5units on the y-axis.

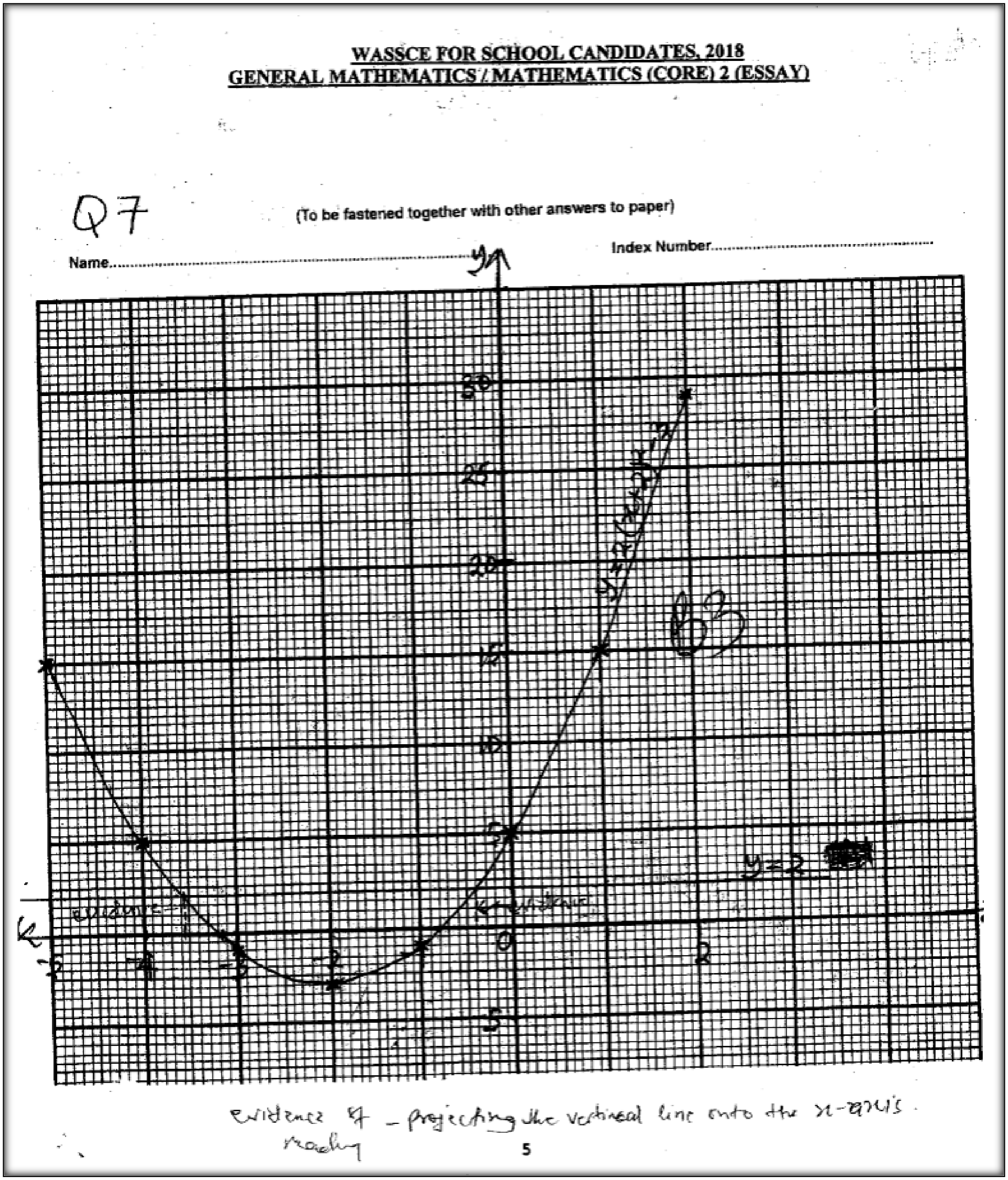
In part (c), most candidates could not read correctly from their graphs the values of x for which (i) $2(x+2)^2 = 3$ and (ii) $2(x+2)^2 = 5$.

In part (d), candidates were required to find the values of x for which y is increasing **.** Most of the candidates who attempted got it wrong. Candidates were expected to answer this question as:

(a)

x	-5	-4	-3	-2	-1	0	1	2
y	15	5	-1	-3	-1	5	15	29

(b) the graph of $y = 2(x + 2)^2 - 3$ for $-5 \leq x \leq 2$

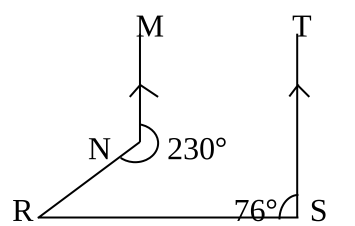


(c)(i) the solution of $2(x + 2)^2 – 3$ is $x = -3.2 \pm(0.1)$ or $-0.8\pm(0.1)$

(ii) $2(x + 2)^2 = 5$ implies $2(x + 2)^2 – 3 = 2$. But $y = 2(x + 2)^2 – 3$ and substituting we have $y = 2$. So next, draw the graph of
$y = 2$ and the solution of $2(x + 2)^2 = 5$ is
$x = -3.6 \pm(0.1)$ or $-0.4 \pm(0.1)$
(d) the values of x for which y is increasing from the graph is $-2 < x \leq 2$

<u>Question 8</u>

(a)



NOT DRAWN TO SCALE

In the diagram, *MN//ST*, $\angle MNR$ = 230° and $\angle TSR$ =76°. Find the value of $\angle NRS$.

(b) (i) Copy and complete the tables for the addition ⊕ and multiplication ⊗ in modulo 5.

⊕	1	2	3	4
1	**2**	**3**	**4**	**0**
2	**3**			
3	**4**			**2**
4	**0**			

⊗	1	2	3	4
1	**1**	**2**	**3**	**4**
2	**2**			
3				**2**
4				**1**

(ii) Use the tables to find:

(α) $4 \otimes 2 \oplus 3 \otimes 4$;

(β) m **such that** $m \otimes m = m \oplus m$;

(γ) n **such that** $3 \oplus n = 2 \otimes n$.

Most of the candidates solved (a) and had the correct answer without showing how it was obtained.

The candidates were required to answer the question as:
Produce $\overline{\text{MN}}$ to meet $\overline{\text{RS}}$ at Q.
Then, $\angle \text{RNQ} = 230^\circ - 180^\circ = 50^\circ$.
Similarly, $\angle \text{NQR} = \angle \text{TSR} = 76^\circ$
$50^\circ + 76^\circ + \angle \text{NRQ} = 180^\circ$
$\angle \text{NRQ} = \angle \text{NRS} = 54^\circ$

In part (b), most of the candidates were able to complete the tables for the addition $\oplus$ and multiplication $\otimes$ in modulo 5 correctly. They were also able to use the table to solve the rest of the questions. Candidates were expected to answer the question as

$\oplus$	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

$\otimes$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

($\propto$) $4 \otimes 2 \oplus 3 \otimes 4 = 3 \oplus 2 = 0$;
(β) $n \otimes m = m \oplus m \Longrightarrow m = 2$;
(γ) $3 \oplus n = 2 \otimes n \Longrightarrow n = 3$.

Question 9

(a) If $16^n = 3\sqrt{2^2}$, find the value of *n*.
(b) The perimeter of a square and a rectangle is the same. The width of the rectangle is 6cm and its area is 16cm^2 less than the area of the square. Find the area of the square.

In part (a), the few candidates who attempted it did not perform well. Candidates were to find the value of *n* if $16^n = \sqrt[3]{2^2}$.
The candidates were expected to answer the question as

$16^n = \sqrt[3]{2^2}$

$2^{4n} = 2^{\frac{2}{3}}$

$4n = \frac{2}{3}$

$n = \frac{1}{6}$

Part (b) was a worded problem and the candidates could not translate the given information into mathematical statements to find the area of the square.

The candidates were expected to answer the question as
Let s = sides of the square, l = length of the rectangle and
w = width of the rectangle.
$4s = 2(l + w)$
$4s = 2(l + 6)$
$l = 2s - 6$………………………………… (1)
$lw = s^2 - 16$ ……………………….……… (2)
substituting for l in equation (2) and $w = 6$,
$6(2s - 6) = s^2 - 16$

$s^2 - 12s + 20 = 0$
(s – 2) (s – 10) = 0
s = 2 or 10
Area of square = 10 × 10
= 100 cm^2

Question 10

The table shows the distribution of marks scored by 500 candidates in an examination.

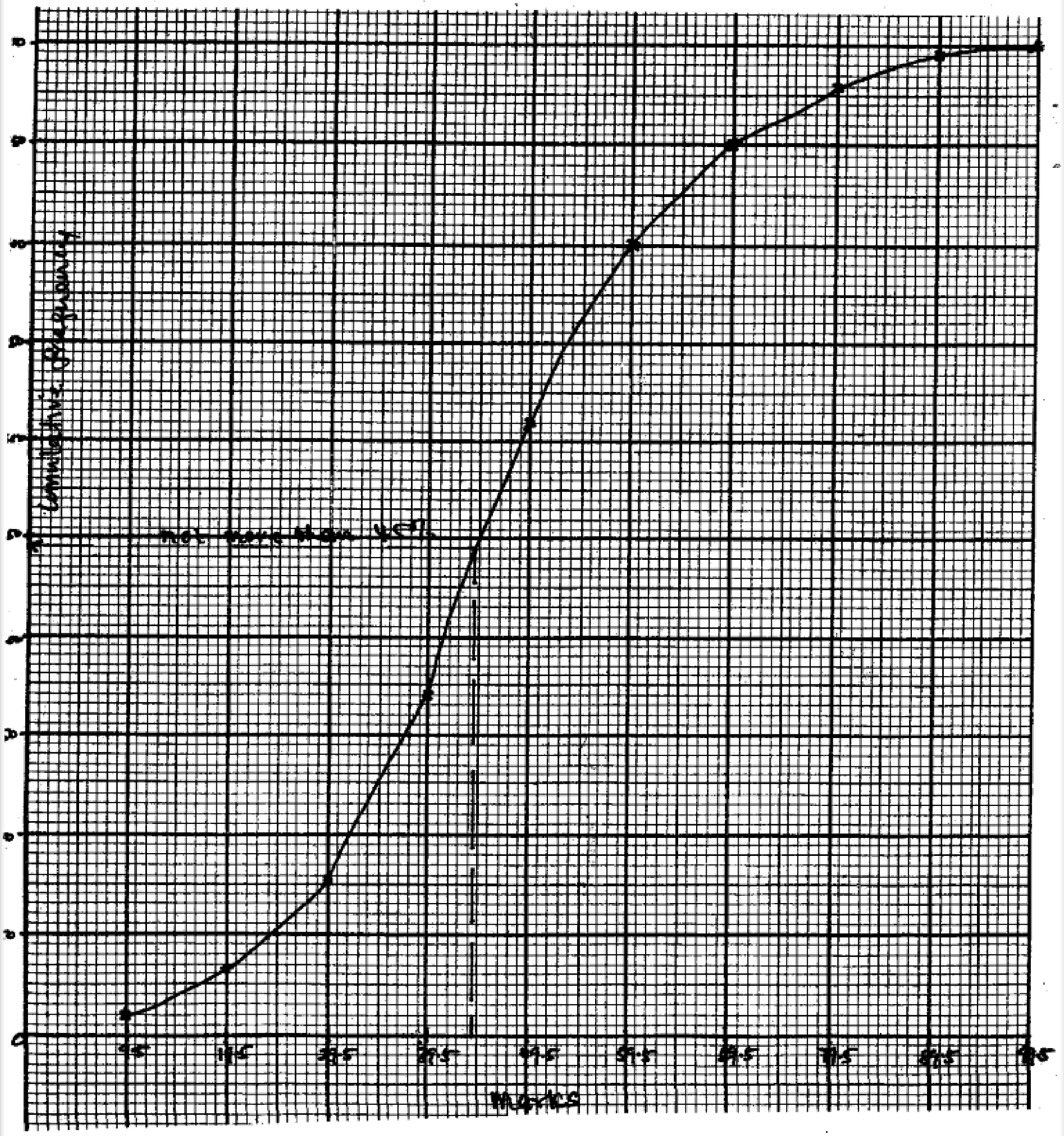
Marks	**0 - 9**	**10 – 19**	**20 - 29**	**30 - 39**	**40 - 49**	**50 - 59**	**60 - 69**	**70 -79**	**80 - 89**	**90 - 99**
Frequency	**10**	**28**	**40**	**92**	**y + 60**	**90**	**50**	**30**	**15**	**5**

(a) Find the value of *y*.
(b) Construct a cumulative frequency table and use it to draw a cumulative frequency curve.
(c) Use the curve to estimate the probability of selecting a candidate who scored not more than 45%.

In part (a), most of the candidates added the frequency correctly. That is
10 + 28 + 40 + 92 + y + 60 + 90 + 50 + 30 + 15 + 5 = 500
y = 80
In part (b), the cumulative frequency table and its corresponding curve were constructed and drawn well by the candidates.
The candiodates were required to solve the question as;
(b)

Upper Class Boundary	Frequency	Cumulative Frequency
9.5	10	10
19.5	28	38
29.5	40	78
39.5	92	170
49.5	140	310
59.5	90	400
69.5	50	450
79.5	30	480
89.5	15	495
99.5	5	500

Q10



8

Scales
upper boundaries

In part (c), a lot of the candidates who attempted this question did not perform well. They were unable to read correctly from the graph, the number of candidates who scored not more than 45% (i.e number of candidates who scored up to 45%). Candidates were expected to answer the (c) as

P(selecting a candidate who scored not more than 45%) = $\frac{240}{500}$

= $\frac{245}{500}$ or $\frac{250}{500}$

= 0.48 or 0.49 or 0.5

Question 11

The angle of elevation of the top, *X*, of a vertical pole from a point, *W*, on the same horizontal grounds as the foot, *Z*, of the pole is 60°. If *W*, is 15km from *X* and 12km from a point *Y* on the pole,

(a) illustrate this information with a diagram;

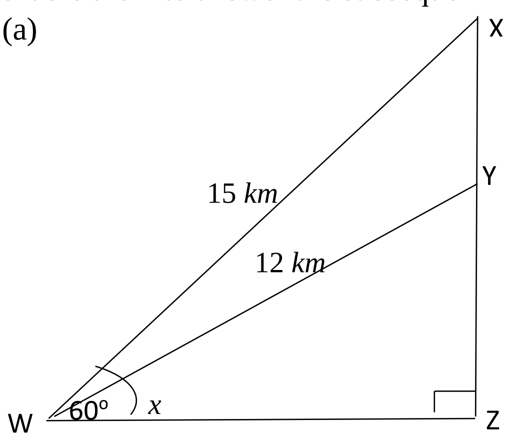
(b) calculate, correct two decimal places, the:

(i) angle of elevation of *Y* from *W;*

(ii) length, *XY*.

Most of the candidates did not attempt this question on trigonometry using the concept of angle of elevation. Those who attempted it could not illustrate the information with a diagram which will enable them to answer the subsequent questions. The required illustration is shown as;

(a)



(b)(i) $\cos 60^\circ = \frac{|WZ|}{15}$

|WZ| = 15 × $\cos 60^\circ$

= 7.5 km

(ii) $\cos x = \frac{7.5}{12} = 0.625$

$x = \cos^{-1}(0.625) = 51.32^\circ$ (to **two** decimal places)

$\sin 60^\circ = \frac{|XZ|}{15}$

|XZ| = 15 × 0.8660 = 12.99 km

|YZ| = 12 × $\sin 51.32^\circ$ = 9.3678 km

$|XY| = 12.99 - 9.3678 = 3.6222$ *km*
≈ 3.62 *km* (to **two** decimal places)

<u>Question 12</u>

(a) Using scales of 2cm to 2 units on both axes, draw on a sheet of graph paper two perpendicular axes O*x* and O*y*, for - 10$\leq x \leq 10$ and – 10 $\leq$ y $\leq$ 10.

(b) Given points *E*(3, 2), $F(-1$, 5) and the vectors $\overrightarrow{FG} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\overrightarrow{GH} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$, find the coordinates of the points *G* and *H*.

(c) Draw, on the same graph, indicating clearly the vertices and their coordinates, the:
  (i) quadrilateral *EFGH*;
  (ii) image $E_1F_1G_1H_1$ of the quadrilateral *EFGH* under an anticlockwise rotation of 90° about the origin where $E \to E_1$, $F \to F_1$, $G \to G_1$ and $H \to H_1$.

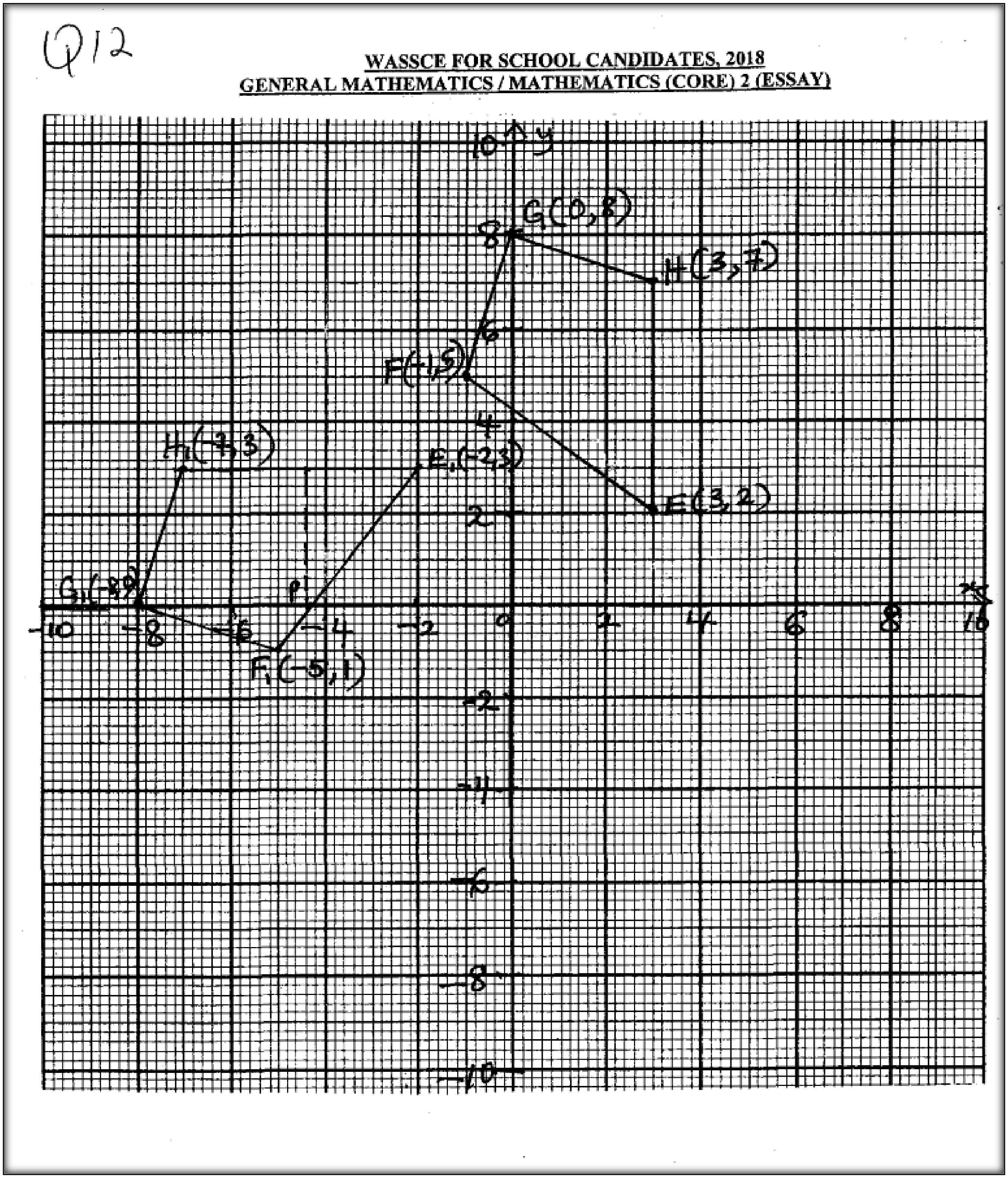
(d) The side E_1F_1 of the quadrilateral $E_1F_1G_1H_1$ cuts the *x*-axis at the point *P*. Calculate, correct to one decimal place, the area of $E_1H_1G_1P$.

In part (a), most of the candidates were able to draw the axes correctly using the given scale. In part (b), candidates were able to find the coordinates of *G* and *H* correctly using vectors $\overrightarrow{OG} = \overrightarrow{OF} + \overrightarrow{FG}$ and $\overrightarrow{OH} = \overrightarrow{OG} + \overrightarrow{GH}$.

In part (c), candidates were able to draw the quadrilateral *EFGH* and indicate the coordinates correctly and they were also able to find its image under the anticlockwise rotation of 90° about the origin.
In part (d), candidates were unable to calculate the area of the $E_1H_1G_1P$ using the appropriate formula.

The candidates were required to solve this question as:

12 (a)



Q12
WASSCE FOR SCHOOL CANDIDATES, 2018
GENERAL MATHEMATICS / MATHEMATICS (CORE) 2 (ESSAY)
y
x
G(0,8)
H(3,7)
F(-1,5)
E(3,2)
H1(-7,3)
E1(-2,3)
F1(-5,1)
P

(b) $\begin{pmatrix}-1\\5\end{pmatrix} + \begin{pmatrix}1\\3\end{pmatrix} = \overline{OG}$

$\begin{pmatrix}0\\8\end{pmatrix} = \overline{OG}$. Then, G(0,8) and H(3,7)

(c) (i) For the quadrilateral EFGH (see the graph)

(ii) For the image of the quadrilateral $E_1(-2, 3)$, $F_1(-5, -1)$, $G_1(-8, 0)$ and $H_1(-7, 3)$ (see the graph)

(d) The lengths of the parallel sides are 4.8 units, 3.7 units and the height is 2.8 units.

Area of $E_1H_1G_1P = \frac{1}{2}(4.8 + 3.7)2.8 = 11.9$ square units

Question 13

(a) Given that P = $\begin{pmatrix}y & 8\\3 & 2\end{pmatrix}$, Q = $\begin{pmatrix}-3 & -5\\-4 & x\end{pmatrix}$, R= $\begin{pmatrix}-56 & -93\\Z & -27\end{pmatrix}$ and PQ = R, find the values of *x*, *y*, and *z*.

(b) (i) (Draw, on a graph paper, using a scale of 2cm to 1 unit on both axes, the lines

$x = 1$;

$y = 2$; and

$x + y = 5$

(ii) Shade the region which satisfies simultaneously the inequalities:

$x + y \leq 5$;

$y \geq 2$ and $x \geq 1$

The candidates who attempted this question could not solve the (a) part correctly. In part (b), candidates could not draw the lines $x = 1$, $y = 2$ and $x + y = 5$ correctly. Neither could they shade the required regions properly. Candidates were expected to answer question (13) as;

(a) $\begin{pmatrix}y & 8\\3 & 2\end{pmatrix}\begin{pmatrix}-3 & -5\\-4 & x\end{pmatrix} = \begin{pmatrix}-3y-32 & -5y+8x\\-9-8 & -15+2x\end{pmatrix}$

$\begin{pmatrix}-3y-32 & -5y+8x\\-9-8 & -15+2x\end{pmatrix} = \begin{pmatrix}-59 & -93\\z & -27\end{pmatrix}$

$-3y - 32 = -59$

$-3y = -27$

$y = 9$

$-15 + 2x = -27$

$2x = -12$

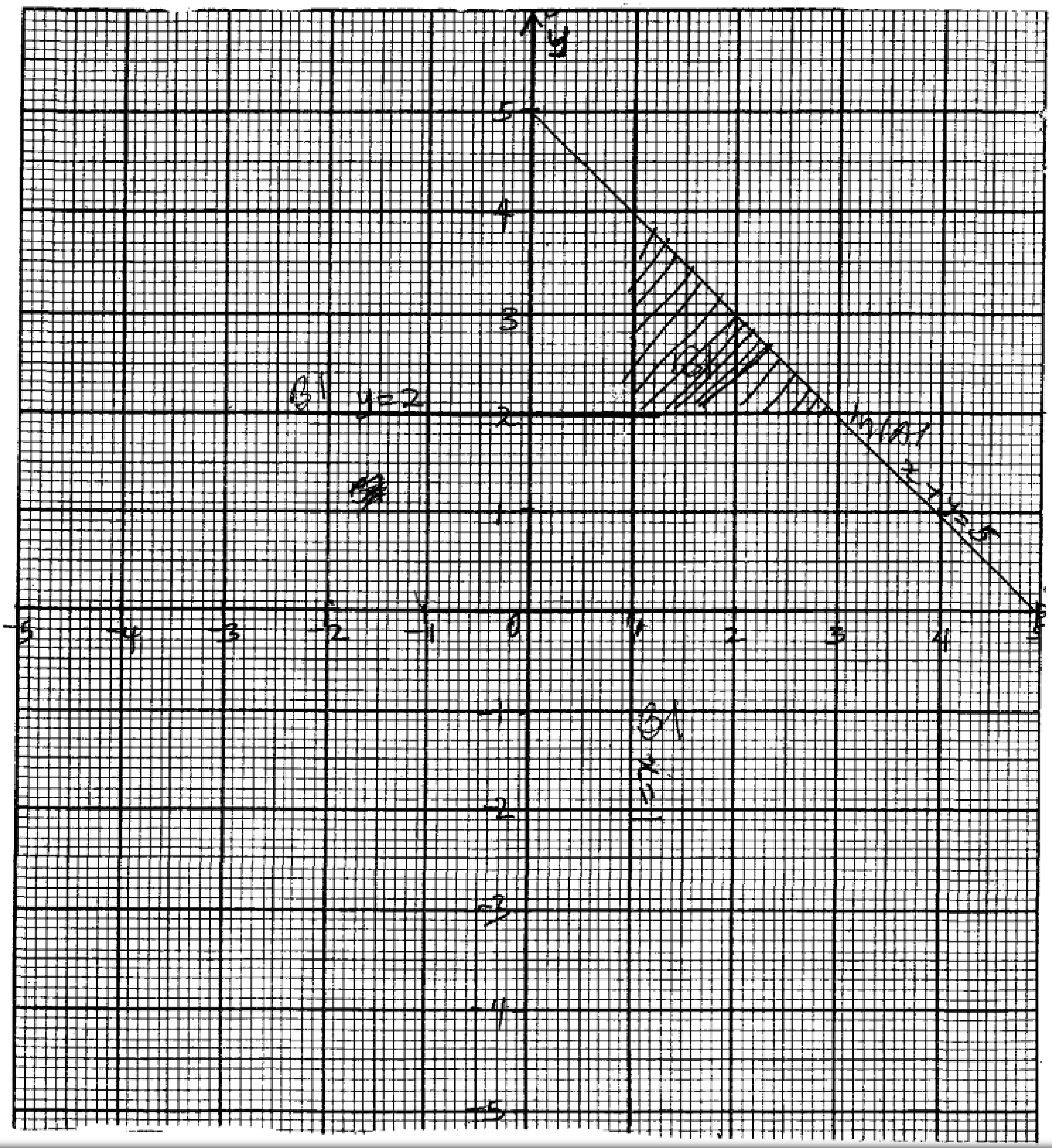
$x = -6$

$-9 - 8 = -z$

$z = -17$

Q 13b

WASSCE FOR SCHOOL CANDIDATES, 2018
GENERAL MATHEMATICS / MATHEMATICS (CORE) 2 (ESSAY)



FURTHER MATHEMATICS/ MATHEMATICS (ELECTIVE)

1. STANDARD OF THE PAPER

The Chief Examiner reported that the standard of the paper compared favourably with that of the previous year.

2. PERFORMANCE OF CANDIDATES

The Chief Examiner reported that there was a decline in the performance of candidates as compared to that of the previous year.

3. SUMMARY OF CANDIDATES' STRENGTHS

The Chief Examiner for Mathematics (Elective) listed some of the strengths of candidates as ability to:

(a) compute the Spearman's Rank correlation coefficient from a given data;

(b) use moment equation to solve related questions;

(c) find acute angle between two given vectors;

(d) apply laws of logarithm to solve related question.

4. SUMMARY OF CANDIDATES WEAKNESSES

The Chief Examiner for Mathematics (Elective) listed some of the weaknesses of candidates as difficulty in:

(a) expressing rational function into partial fraction;

(b) finding equation of a circle passing through three points;

(c) finding the nth term from the sum of the *n* terms of a sequence;

(d) solving probability related problems.

5. SUGGESTED REMEDIES

The Chief Examiner for Mathematics (Elective) suggested that teachers should give students sufficient exercises for the various topics treated with the students. He also recommended that teachers should give in-depth tuition in areas of students' weaknesses by explaining thoroughly the relevant concepts.

6. DETAILED COMMENTS

Question 1.

Given that A = $\begin{pmatrix} \mathbf{3m+2} & \mathbf{10} \\ \mathbf{4m+2n} & \mathbf{5} \end{pmatrix}$**, B =** $\begin{pmatrix} \mathbf{14} & \mathbf{3m-n} \\ \mathbf{20} & \mathbf{5} \end{pmatrix}$ **and A ≡ B,**
(a) find the values of *m* and *n*;
(b) write the matrices A and B.

The candidates were required to equate corresponding entries of the two matrices and solve for ***m*** and ***n***. In part (b), the candidates were required to write down the matrices **A** and **B**.

Some of the candidates who attempted the (a) part of the question were able to equate the corresponding entries to solve for ***m*** and ***n*** correctly. Some of them could not write down the matrices **A** and **B** in (b).
Candidates were expected to answer question (1) as

(a) $\begin{pmatrix} 3m+2 & 10 \\ 4m+2n & 5 \end{pmatrix} = \begin{pmatrix} 14 & 3m-n \\ 20 & 5 \end{pmatrix}$

$3m + 2 = 14$

$3m = 12$

$m = 4$

$4 \times 4 + 2n = 20$

$16 + 2n = 20$

$2n = 4$

$n = 2$

(b) A = $\begin{pmatrix} 14 & 10 \\ 20 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 14 & 10 \\ 20 & 5 \end{pmatrix}$

Question 2

Solve the equations: log (*x*−1) + 2log*y* = 2log 3;
log *x* + log*y* = log 6.

The candidates were expected to apply the laws of logarithms to generate two simultaneous equations and solve for the values of *x* and ***y*** simultaneously.

Majority of the candidates were able to apply the laws of logarithm correctly to generate the two equations which enabled them to solve for *x* and *y*.

The candidates were expected to answer the question as log (x -1) + 2log y = 2log3

log [y^2 (x -1)] = log 9
y^2 (x-1) = 9
$y^2x - y^2 = 9$…………………………………...……... (1)
log x + log y = log 6
log xy = log 6
$xy = 6$………………………………………………... (2)
From eqns (1) and (2)
$6y - y^2 = 9$
$y^2 - 6y + 9 = 0$
$y(y-3) - 3(y-3) = 0$
$(y-3)^2 = 0$
$y = 3$ ……………………………………….………... (3)
From equations (2) and (3)
$3x = 6$
$x = 2$

Question 3

Express $\frac{1}{x^2\,(2x-1)}$ in partial fractions.

Candidates were required to express $\frac{1}{x^2\,(2x-1)}$ in partial fractions. Most candidates did not realize that the denominator was linear and repeated factors of the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$

The candidates were expected to answer question (3) as:

$\frac{1}{x^2(2x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$

$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$

$1 \equiv 2Ax^2 - Ax + 2Bx - B + Cx^2$

$1 \equiv x^2(2A + C) - (A - 2B)x - B$

$B = -1$

$A - 2(-1) = 0$

$A + 2 = 0$

$A = -2$

$2(-2) + C = 0$

$C = 4$

$\frac{1}{x^2(2x-1)} \equiv -\frac{2}{x} - \frac{1}{x^2} + \frac{4}{2x-1}$

Question 4.

The sum of the first n terms of a sequence is given by $S_n = \frac{5}{2}n^2 + \frac{5}{2}n$.
Write down the first four terms of the sequence and find the nth term of the sequence.

The candidates were given the sum of the first nth terms of a sequence and were required to write down the first four terms and find the nth term of the sequence. Majority of the candidates who attempted this question could not tell the difference between the terms of the sequence and the sum of the terms. For example, S_1, S_2, S_3 and S_4 were taken to be U_1, U_2, U_3 and U_4 respectively and they could not have proceeded to find Un.

The candidates were expected to answer the question as:

$S_n = \frac{5n^2}{2} + \frac{5n}{2}$

1^{st} term = 5
2^{nd} term = 10
3^{rd} term = 15
4^{th} term = 20 or
5,10,15,20
5 + 5+ d = 15
d = 5
u_n (nth term) = 5 + 5(n-1)
= 5 + 5n – 5
u_n = 5n

Question 5

The table shows the relation between weekly advertisement (x) and number of items sold (y) in a shop.

x	**0**	**3**	**6**	**9**	**2**	**4**	**8**	**1**	**5**	**7**
y	**0**	**25**	**63**	**94**	**105**	**12**	**42**	**81**	**101**	**107**

Calculate the Spearman's rank correlation coefficient.

Candidates were expected to calculate the Spearman's rank correlation coefficient from a relationship between weekly advertisement (x) and number of items sold (y) in a shop.

A lot of the candidates were able to rank the (x) and the (y) correctly and they used the correct formula to calculate the rank correlation coefficient.

The candidates were required to answer the question as:

x	y	x_r	y_r	d^2
0	0	10	10	0
3	25	7	8	1
6	63	4	6	4
9	94	1	4	9
2	105	8	2	36
4	12	6	9	9
8	42	2	7	25
1	81	9	5	16
5	101	5	3	4
7	107	3	1	4
				108

$n = 10$

$r = 1 - \frac{6\ x\ 108}{10\ (100-1)}$

$= 1 - \frac{648}{990}$

$= \frac{19}{55}$ or 0.34545

<u>Question 6</u>

The probabilities that Sani, Kato and Titi will hit a target are $\frac{3}{4}$, $\frac{2}{5}$ and $\frac{1}{3}$ respectively. If all the three men shoot once, what is the probability that the target will be hit only once.

The candidates were expected to find the probability that the target will be hit only once given the probability that Sani, Kato and Titi will hit the target. A lot of the candidates who attempted this question were not able to find the probability that Sani, Kato and Titi will not hit the target and could not proceeded to find the correct answer.

Candidates were expected to answer question (6) as:

P_1 (Sani will hit target) = $\frac{3}{4}$

P (Sani will not hit target) = $\frac{1}{4}$

P(Kato will hit target) = $\frac{2}{5}$

P(Kato will not hit target) = $\frac{3}{5}$

P(Titi will hit target) = $\frac{1}{3}$

P(Titi will not hit target) = $\frac{2}{3}$

P(Target will be hit only once)

$=\left(\frac{3}{4} \times \frac{3}{5} \times \frac{2}{3}\right) + \left(\frac{2}{5} \times \frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{1}{3} \times \frac{1}{4} \times \frac{3}{5}\right)$

$= \frac{18}{60} + \frac{4}{60} + \frac{3}{60}$

$= \frac{25}{60}$

$= \frac{5}{12}$ or 0.4167

Question 7

Given that p = 2i + 3j, q = 3i - 2j and r = -i + 6j, find, $|6\boldsymbol{p} - 3\boldsymbol{q} - 4\boldsymbol{r}|$.

The candidates were required to use the given vectors **p** = 2**i** + 3**j**, **q** = 3**i** - 2**j** and **r** =−**i** + 6**j** to find $|6\boldsymbol{p} - 3\boldsymbol{q} - 4\boldsymbol{r}|$. Most of the candidates who attempted this question were able to make correct vector substitutions and found the magnitude of the vector $6\boldsymbol{p} - 3\boldsymbol{q} - 4\boldsymbol{r}$ correctly.

Candidates were required to answer this question as

P = 2**i**+3**j**, **q** = 3**i** – 2**j**, **r** = −**i**+6**j**

6**p** – 3**q** – 4**r** = $6\begin{pmatrix}2\\3\end{pmatrix} - 3\begin{pmatrix}3\\-2\end{pmatrix} - 4\begin{pmatrix}-1\\6\end{pmatrix}$

$= \begin{pmatrix}12\\8\end{pmatrix} + \begin{pmatrix}-9\\6\end{pmatrix} + \begin{pmatrix}4\\-24\end{pmatrix}$

$= \begin{pmatrix}7\\0\end{pmatrix}$ or 7**i**

|6**p** – 3**q** – 4**r**| = $\sqrt{49}$

= 7

Question 8

A body of mass 3 kg moving at 15 ms^{-1} collides with another body of mass 5 kg moving at 26 ms^{-1} in the same direction. After collusion the 5kg body moves at 39. 8ms^{-1} in the same direction. Find the velocity of the 3 kg body after the collision.

The candidates were given two bodies with masses and velocities to find the velocity of the 3kg body after a collision. Candidates were able to write the correct formula, made the right substitutions to arrive at the correct solution. However, few candidates failed to state the direction of the 3kg body after the collision. Candidates were therefore expected to answer the question as

$3 \times \boldsymbol{v} + 5 \times 39.8 = 3 \times 15 + 5 \times 26$

$3\boldsymbol{v} + 199 = 175$

$\boldsymbol{v} = \frac{175-199}{3}$

$= -8ms^{-1}$

Final velocity of 3kg body = $8ms^{-1}$ in the opposite direction

Question 9

The points (2,5) (3,0) and (10, -13) lie on a circle. Find:

- **(a) the coordinates of the centre of the circle;**
- **(b) the equation of the circle;**
- **(c) the radius of the circle.**

The candidates were expected to substitute the three points into the general equation of a circle. Majority of the candidates after substituting the points into the equation could not simplify to find the centre, the equation and the radius of the circle. Only very few candidates were able to solve the three resulting equations simultaneously to enable them to answer all the three parts (a), (b) and (c) correctly.

The candidates were expected to answer the question as:

(a) $x^2 + y^2 + 2gx + 2fy + k = 0$

For point (2,5),

$4 + 25 + 4g + 10f + k = 0$

$4g + 10f + k = -29$ …………..... Eqn (1)

For point (3, 0)

$9 + 0 + 6g + 0 + k = 0$

$6g + k = -9$ …………………… Eqn (2)

For point (10,−13)

$100 + 169 + 20g - 26f + k = 0$

$20g - 26f + k = -269$ ………….. Eqn (3)

From equation (1) and (2)

$4g + 10f + (-9 - 6g) = -29$

$-2g + 10f = -20$ ……………… Eqn (4)

Eqn (3) $20g - 26f + (-9 - 6g) = -269$

$14g - 26f = -260$ ……………… Eqn (5)

Solving equations (4) and (5)

$-14g + 70f = -140$

$\underline{14g - 26f = -260}$

$44f = -400$

$f = -\frac{100}{11}$

$-2g - \frac{1000}{11} = -20$

$-22g - 1000 = -220$

$-22g = -780$

$g = -\frac{780}{-22}$

$= \frac{390}{11}$

$6\left(\frac{390}{11}\right) + k = -9$

$k = \frac{-99}{11} - \frac{2340}{11}$

$= -\frac{2439}{11}$

Coordinates of centre

$= \left(-\frac{390}{11}, \frac{100}{11}\right)$

(b) $x^2 + y^2 + \frac{780}{11}x - \frac{200}{11}y + \frac{2349}{11} = 0$

$11x^2 + 11y^2 + 780x - 200y + 2349 = 0$

(c) radius = $\sqrt{\left(\frac{390}{11}\right)^2 + \left(\frac{-100}{11}\right)^2 - \frac{2349}{11}}$

= 33.558units

Question 10

(a) Solve: $2^{3n+2} - 7 \times 2^{2n+2} - 31 \times 2^n - 8 = 0, n \in \mathbb{R}$.

(b) Find $\int (x^2 + 1)^{\frac{1}{2}} x \, dx$.

The candidates were required to apply the laws of indices to express the part (a) into a polynomial function and solve for *x*. In part (b), candidates were expected to use substitution method to integrate the function given. In part (a), a lot of candidates were not able to apply the laws of indices and the substitution method correctly.

Candidates were required to answer the question as

(a) $2^{3n+2} - 7 \times 2^{2n+2} - 31 \times 2^n - 8 = 0$

$2^2 \times 2^{3n} - 7 \times 2^2 . 2^{2n} - 31 \times 2^n - 8 = 0$

$4 \times (2^n)^3 - 28(2^n)^2 - 31 \times 2^n - 8 = 0$

Let $2^n = y$

$4y^3 - 28y^2 - 31y - 8 = 0$

$(y - 8)(4y^2 + 4y + 1) = 0$

$(y - 8)(2y + 1)(2y + 1) = 0$

$y = 8, -\frac{1}{2}, -\frac{1}{2}$

$2^n = 8$

$n = 3$

(b) Let $u = x^2 + 1$

then $du = 2x\,dx$

$\int (x^2 + 1)^{\frac{1}{2}} . x dx = \frac{1}{2} \int u^{\frac{1}{2}} du$

$= \frac{1}{3} u^{\frac{3}{2}} + c$

$= \frac{1}{3} (x^2 + 1)^{\frac{3}{2}} + c$

<u>Question 11</u>

(a) (i) Find the first four terms of the binomial expansion of $(1 - 2x)^6$ in ascending powers of x.
(ii) Using the expression in (a)(i), calculate, correct to four decimal places, the value of $(0.98)^6$.

(b) If $x^2 + 2x - 8$ is a factor of the polynomial $f(x) = ax^3 - 4x^2 + 28x - 16$, find the value of a.

The candidates were expected to use binomial expansion to expand the expression $(1-2x)^6$ to obtain the first four terms and use the expression to find the value of $(0.98)^6$ in (a). In part (b), candidates were required to substitute the factor into the given polynomial to find the value of ***a***. In part (a), most of the candidates were able to obtain the first four terms of the expansion but could not find the value of $(0.98)^6$ using the expansion. In part (b), few of the candidates were able to substitute the factor into the given polynomial to obtain the correct answer.
Candidates were expected to answer question (11) as:

(a) (i) $(1-2x)^6 = 1 + 6(-2x) + \frac{6\times5}{1\times2}(-2x)^2 + \frac{6\times5\times4\,(-2x)^3}{1\times2\times3}$

$= 1 - 12x + 60x^2 - 160x^3$

$(0.98)^6 = \{1 - 2(0.01\}^6$

$x = 0.01$

(ii) $(0.98)^6 = 1 - 12(0.01) + 60(0.01)^2 - 160(0.01)^3$

$= 0.88584$

$= 0.8858$

(b) $f(x) = ax^3 - 4x^2 + 28x - 16$

$x^2 + 2x - 8 = 0$

$x^2 + 4x - 2x - 8 = 0$

$x(x + 4) - 2(x + 4) = 0$

$(x + 4)(x - 2) = 0$

$x = -4, x = 2$

for $x = 2$,

$a(2^3) - 4(2)2 + 28(2) - 16 = 0$

$8a - 16 + 56 - 16 = 0$

$8a + 24 = 0$

$8a = -24$

$a = -3$

Question 12

Ten coins were tossed together a number of times and the distribution of the number of tails obtained is given in the table below:

Number of tails	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Frequency	**3**	**8**	**24**	**35**	**10**	**60**	**101**	**11**	**9**	**4**	**3**

Calculate, correct to two decimal places, the:

(a) mean number of tails;
(b) probability of obtaining an odd number of tails;
(c) probability of obtaining an even number of tails.

The candidates were expected to use the frequency table to answer questions on (a) the mean number of tails, (b) probability of obtaining an odd number of tails and probability of obtaining an even number of tails in (c). Majority of candidates who attempted this question were able to

answer only the (a) part correctly and most of the candidates were not able to find the mean in (a), probability of obtaining an odd and even number of tails in (b) and (c).
Candidates were expected to answer question (12) as:

(a)

x	f	fx	
0	3	0	
1	8	8	
2	24	48	
3	35	105	
4	10	40	
5	60	300	
6	101	606	
7	11	77	
8	9	72	
9	4	36	
10	3	30	
	268	1322	

Mean $\frac{1322}{268}$

= 4.93

(b) P(obtaining odd numbers of tails)

$= \frac{8+35+60+11+4}{268}$

$= \frac{118}{268}$

= 0.44

(c) P (obtaining even numbers of tails)

$= \frac{3+24+10+10+101+9+3}{268}$

= 0.56

Question 13

The probabilities that Ato, Sulley and Kofi will gain admission to a certain university are $\frac{4}{5}$, $\frac{3}{4}$ and $\frac{2}{3}$ respectively. Find the probability that:

(a) none of them will gain admission;
(b) at most two of them will gain admission;
(c) only Kofi and Ato will gain admission.

Candidates were expected to use the probability that Ato, Sulley and Kofi will gain admission to university to find the probability that (a) none of them will gain admission, (b) two of them will gain admission and (c) only Kofi will gain admission. Few of the candidates were able to solve for (a) and (b) correctly. Most of the candidates were not able to solve the part (c) of the question.

Candidates were expected to answer question (13) as:

P (Ato will gain admission) = $\frac{4}{5}$

P (Ato will not gain admission) = $\frac{1}{5}$

P (Sulley will gain admission) = $\frac{3}{4}$

P (Sulley will not gain admission) = $\frac{1}{4}$

P (Kofi will gain admission) = $\frac{2}{3}$

P (Kofi will not gain admission) = $\frac{1}{3}$

(a) P (none will gain admission)

$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{60}$

(b) P (exactly one will gain admission)

$= \left(\frac{4}{5} \times \frac{1}{4} \text{ x} \frac{1}{3}\right) + \left(\frac{1}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{1}{5} \times \frac{1}{4} \times \frac{2}{3}\right)$

$= \frac{4}{60} + \frac{3}{60} + \frac{2}{60}$

$= \frac{9}{60}$

P (exactly two will gain admission)

$= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}\right)$

$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60}$

$= \frac{26}{60}$

P (at most two will gain admission)

$= \frac{1}{60} + \frac{9}{60} + \frac{26}{60}$

$= \frac{36}{60}$

$= \frac{3}{5}$

(c) P(only Kofi and Ato)

$= \left(\frac{2}{3} \times \frac{1}{4} \times \frac{4}{5}\right)$

$\frac{8}{60}$

$\frac{2}{15}$

<u>Question 14</u>

The position vectors of points *P*, *Q* and *R* with respect to the origin are (4i −5j), (i + 3j) and (−5i + 2j) respectively. If *PQRM* is a parallelogram, find:

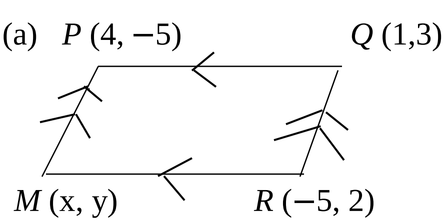
(a) the position vector of *M*;

(b) $|\overrightarrow{PM}|$ and $|\overrightarrow{PQ}|$;

(c) the acute angle between $\overrightarrow{PM}$ and $\overrightarrow{PQ}$, correct to one decimal place.

Candidates were required to use the parallelogram *PQRM* with the position vectors *P*, *Q* and *R* with respect to the origin to find (a) the position vector of *M*, (b) find $|\overrightarrow{PM}|$, and $|\overrightarrow{PQ}|$ and (c) acute angle between $|\overrightarrow{PM}|$and $|\overrightarrow{PQ}|$. Most of the candidates were able to find the position vector in (a), $|\overrightarrow{PM}|$ and $|\overrightarrow{PQ}|$ in (b) and (c) acute angle between $|\overrightarrow{PM}|$and $|\overrightarrow{PQ}|$. Candidates were expected to answer question (14) as

(a)



$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ 8 \end{pmatrix}$

$\overrightarrow{MR} = \begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

$= \begin{pmatrix} -5 - x \\ 2 - y \end{pmatrix}$

$\begin{pmatrix} -3 \\ 8 \end{pmatrix} = \begin{pmatrix} -5 & -x \\ 2 & -y \end{pmatrix}$

$-5 - x = -3 \Longrightarrow x = -2$

$2 - y = 8 \Longrightarrow y = -6$

The position vector of M

$= -2\mathbf{i} - 6\mathbf{j}$

(b) $\overrightarrow{PM} = (-2\mathbf{i} - 6\mathbf{j}) - (4\mathbf{i} - 5\mathbf{j})$

$= -6\mathbf{i} - \mathbf{j}$

$|\overrightarrow{PM}| = \sqrt{36 + 1}$

$= \sqrt{37}$ units

$\overrightarrow{PQ} = (\mathbf{i} + 3\mathbf{j}) - (4i - 5j)$

$= -3\mathbf{i} + 8\mathbf{j}$

$|\overrightarrow{PQ}| = \sqrt{9 + 64}$

$= \sqrt{73}$ units

(c) $\overrightarrow{PQ} = -3\mathbf{i} + 8\mathbf{j}$, $\overrightarrow{PM} = -6\mathbf{i} - \mathbf{j}$

$\overrightarrow{PQ} \cdot \overrightarrow{PM} = 18 - 8$

$\overrightarrow{PQ} \cdot \overrightarrow{PM} = 10$

$\text{Cos } \theta = \frac{10}{\sqrt{37} \, x \, \sqrt{73}}$

$\theta = \cos^{-1}\left(\frac{10}{51.97}\right)$

$\theta = 78.9°$

Question 15

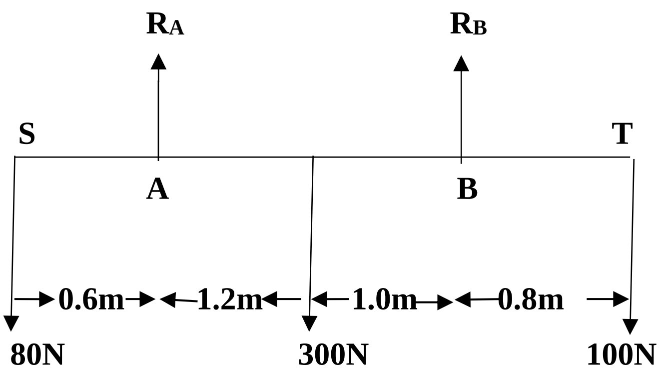
(a) A uniform beam *ST* of length 3.6 m and mass 30kg rests on two pivots *A* and *B*, such that $|SA|$ = 0.6m and $|TB|$ = 0.8m. Loads of 8 kg and 10 kg are attached at *S* and *T* respectively. Find, correct to two decimal places, the reactions at *A* and *B*. [Take g = 10 ms^{-2}]

(b) A car starts from rest and accelerates at $8ms^{-2}$ for 10 seconds. It then continues at the same velocity for 20 seconds. Calculate the total distance travelled.

The candidates were expected to find the reactions of A and B given a uniform beam *ST* of length 3.6m and mass 30kg resting on two pivots *A* and *B*, such that $|SA|$ = 0.6m and $|TB|$ = 0.8m. Loads of 8 kg and 10 kg are attached at *S* and *T* respectively and (b) calculate the total distance given the acceleration, time and the velocity. Majority of the candidates did not attempt the part (a) of this question and those who attempted could not draw the diagram correctly. In (b), very few of the candidates were able to write the formula for finding the total distance and made correct substitutions to obtain the right solution.

Candidates were required to answer question (15) as:

(a)



R_B x 2.2 + 48 = 360 + 300

R_B x 2.2 + 48 = 360 + 300

$R_{B1} = \frac{660-48}{2.2}$

= 278.18N

$R_A + R_B$ = (80 + 300 + 100)N

= 480N

R_A = 480 – 278.18

= 201. 82N

(b) V = 8 x 10

= 80ms^{-1}

$S_1 = \frac{1}{2}$ x 8 x 10^2

= 400m

S_2 = 80 x 20

= 1600m

Total distance = 400 + 1600

= 2000m